

A

DISCOURSE

Concerning GRAVITY, and its Properties, wherein the Descent of Heavy Bodies, and the Motion of Projects is briefly, but fully handled: Together with the Solution of a Problem of great Use in GUNNERY. By E. HALLEY.

NATURE amidst the great variety of *Problems* wherewith She exercises the Wits of Philosophical men, scarce affords any one wherein the Effect is more visible, and the Cause more concealed than in those of the *Phænomena* of Gravity. Before we can go alone, we must learn to defend our selves from the violence of its Impulse, by not trusting the *Center of Gravity* of our Bodies beyond our reach; and yet the Acutest Philosophers, and the subtlest Enquirers into the Original of this Motion, have been so far from satisfying their Readers, that they themselves seem little to have understood the Consequences of their own *Hypotheses*.

Des Cartes his Notion, I must needs confess to be to me Incomprehensible, while he will have the Particles of his *Celestial matter*, by being reflected on the Surface of the *Earth*, and so ascending therefrom, to drive down into their places those *Terrestrial Bodies* they find above them: This is as near as I can gather the scope of the 20, 21, 22, and 23 *Sections* of the last Book of his *Principia Philosophiæ*; yet neither he, nor any of his Followers can shew how a Body suspended *in libero athere*, shall be carried downwards by a

A

conti-

continual Impulse tending upwards, and acting upon all its parts equally: And besides the obscurity wherewith he expresses himself particularly, *Seet. 23.* does sufficiently argue according to his own Rules, the confused *Idea* he had of the thing he wrote.

Others, and among them Dr. *Vossius* asserts the Cause of the *Descent* of heavy Bodies to be the *Diurnal rotation* of the *Earth* upon its *Axis*, without considering, that according to the Doctrine of Motion fortified with Demonstration, all Bodies moved *in Circulo*, would recede from the Center of their Motion; whereby the contrary to *Gravity* would follow, and all loose Bodies would be cast into the Air in a *Tangent* to the *Parallel* of *Latitude*, without the intervention of some other Principle to keep them fast, such as is that of *Gravity*. Besides the effect of this Principle is throughout the whole Surface of the Glob found nearly equal, and certain *Experiment* seems to argue it rather less near the *Equinoctial*, than towards the *Poles*, which could not be by any means, if the *Diurnal rotation* of the *Earth* upon its *Axis* were the cause of *Gravity*, for where the Motion was swiftest, the Effect would be most considerable.

Others assign the *Pressure* of the *Atmosphere*, to be the Cause of this Tendency towards the *Center* of the *Earth*; but unhappily they have mistaken the *Cause* for the *Effect*, it being from undoubted Principles plain, that the *Atmosphere* has no other *Pressure* but what it derives from its *Gravity*, and that the *Weight* of the upper parts of the *Air*, pressing on the lower parts thereof, do so far bend the Springs of that *Elastick Body*, as to give it a force equal to the *Weight* that Compressed it, having of it self no force at all: And supposing it had, it will be very hard to explain the *Modus*, how that *Pressure* should occasion the *Descent* of a *Body* circumscribed by it, and pressed equally above and below, without some other force to draw, or thrust it downwards. But to demonstrate the contrary of this Opinion, an *Experiment* was long since shewn before the *Royal Society*, whereby it appeared

peared that the *Atmosphere* was so far from being the Cause of *Gravity*, that the Effects thereof were much more Vigorous where the *pressure* of the *Atmosphere* was taken off; for a long *Glass-Receiver* having a light Down-feather included, being *evacuated* of *Air*, the Feather which in the *Air* would hardly sink, did *in vacuo* descend with nearly the same *Velocity* as if it had been a Stone.

Some think to Illustrate this *Descent* of *Heavy Bodies*, by comparing it with the Vertue of the *Loadstone*; but setting aside the difference there is in the manner of their *Attractions*, the *Loadstone* drawing only *in* and *about* its *Poles*, and the *Earth* near equally in all parts of its *Surface*, this Comparison avails no more than to explain *ignotum per aque ignotum*.

Others assign a certain *Sympathetical attraction* between the *Earth* and its *Parts*, whereby they have, as it were, a desire to be united, to be the Cause we enquire after: But this is so far from explaining the *Modus*, that it is little more; than to tell us in other terms, that *heavy bodies descend*, because they *descend*.

This I say, not that I can pretend to substitute any *Solution* of this Important *Philosophical Problem*, that shall more happily explicate the *Appearances* of *Gravity*, only it may be serviceable to those with whom the Credit of great *Authors* sways much, and who too-readily assent *in verba magistri*, to let them see that their Books are not always infallible: Besides the detection of Errors is the first and surest step towards the discovery of Truth.

Tho' the Efficient Cause of *Gravity* be so obscure, yet the final Cause thereof is clear enough; for it is by this single *Principle* that the *Earth* and all the *Celestial Bodies* are kept from *dissolution*: the least of their *Particles* not being suffered to recede far from their *Surfaces*, without being immediately brought down again by vertue of this *Natural tendency*, which for their Preservation, the Infinite Wisdom of their *Creator* has Ordained to be towards each of their *Centers*;

nor can the *Globes* of the *Sun* and *Planets* otherwise be destroyed, but by taking from them this power of keeping their parts united.

The Affections or Properties of *Gravity*, and its manner of acting upon *Bodies falling*, have been in a great measure discovered, and most of them made out by *Mathematical demonstration* in this our *Century*, by the accurate diligence of *Galileus*, *Toricellius*, *Hugenius*, and others, and now lately by our worthy Country-man Mr. *Jsaac Newton*, (who has an incomparable *Treatise* of *Motion* almost ready for the *Press*) which Properties it may be very material here to enumerate, that they may serve for a Foundation to all those that shall be willing to spend their Thoughts in search of the true Cause of this *descent* of *Bodies*.

The first Property is, That by this principle of *Gravitation*, all Bodies do descend towards a Point, which either is, or else is very near to the Center of *Magnitude* of the *Earth* and *Sea*, about which the *Sea* forms it self exactly into a *Spherical surface*, and the *Prominences* of the *Land*, considering the Bulk of the whole, differ but insensibly therefrom.

Secondly, That this Point or Center of *Gravitation*, is fixt within the *Earth*, or at least has been so, ever since we have any *Authentick History*: For a Consequence of its Change, tho' never so little, would be the over-flowing of the low Lands on that side of the *Globe* towards which it approached, and the leaving new Islands bare on the opposite side, from which it receded; but for this Two thousand years it appears, that the low Islands of the *Mediterranean Sea* (near to which the ancientest Writers lived) have continued much at the same height above the Water, as they now are found; and no *Inundations* or *Recesses* of the *Sea* arguing any such Change, are Recorded in History; excepting the *Universal Deluge*, which can no better way be accounted for, than by supposing this Center of *Gravitation* removed for a time, towards the middle of the then inhabited parts of the World;
and

and a change of its place, but the two thousandth part of the *Radius* of this *Globe*, were sufficient to bury the Tops of the highest Hills under water.

Thirdly, That in all parts of the *Surface* of the *Earth*, or rather in all Points equidistant from its *Center*, the force of *Gravity* is nearly equal; so that the length of the *Pendulum* vibrating *seconds of time*, is found in all parts of the World to be very near the same. 'Tis true at *S. Helena* in the *Latitude* of 16 Degrees *South*, I found that the *Pendulum* of my Clock which vibrated *seconds*, needed to be made shorter than it had been in *England* by a very sensible space, (but which at that time I neglected to observe accurately) before it would keep time; and since the like Observations has been made by the *French Observers* near the *Equinoctial*: Yet I dare not affirm that in mine it proceeded from any other Cause, than the great height of my place of Observation above the *Surface* of the *Sea*, whereby the *Gravity* being diminished, the length of the *Pendulum* vibrating *seconds*, is proportionably shortned.

Fourthly, That *Gravity* does equally affect all *Bodies*, without regard either to their *matter*, *bulk*, or *figure*; so that the Impediment of the *Medium* being removed, the most compact and most loose, the greatest and smallest *Bodies* would descend the same *spaces* in equal times; the truth whereof will appear from the *Experiment* I before cited. In these two last particulars, is shewn the great difference between *Gravity* and *Magnetism*, the one affecting only *Iron*, and that towards its *Poles*, the other all *Bodies* alike in every part. As a *Corollary*; from hence it will follow, that there is no such thing as *positive levity*, those things that appear light, being only comparatively so; and whereas several things rise and swim in *fluids*, 'tis because bulk for bulk, they are not so heavy as those *fluids*; nor is there any reason why *Cork*, for instance, should be said to be light because it swims on *Water*, any more than *Iron* because it swims on *Mercury*.

Fifthly, That this power encreases as you descend, and de-

decreases as you ascend from the Center, and that in the proportion of the *Squares* of the *distances* therefrom *reciprocally*, so as at a double distance to have but a quarter of the force ; this property is the principle on which Mr. *Newton* has made out all the *Phænomena* of the *Celestial Motions*, so easily and naturally that its truth is past dispute. Besides that, it is highly rational, that the *attractive* or *gravitating* power should exert it self more vigorously in a small Sphere, and weaker in a greater, in proportion as it is contracted or expanded, and if so, seeing that the *surfaces* of *Spheres* are as the *Squares* of their *Radii*, this power at several distances will be as the *Squares* of those *distances Reciprocally*, and then its whole action upon each *Spherical Surface*, be it great or small will be always equal. And this is evidently the rule of *Gravitation* towards the *Centers* of the *Sun*, *Jupiter*, *Saturn* and the *Earth*, and thence is reasonably inferred, to be the general principle observed by *Nature*, in all the rest of the *Celestial Bodies*.

These are the principal affections of *Gravity*, from which the rules of the *fall* of *Bodies*, and the *motion* of *Projects* are *Mathematically* deducible. Mr. *Isaac Newton* has shewed how to define the spaces of the *descent* of a *Body*, let fall from any given highth, down to the *Center*. Supposing the *Gravitation* to increase, as in the fifth Property ; but considering the smallness of highth, to which any *Project* can be made ascend, and over how little an *Arch* of the *Globe* it can be cast by any of our *Engines*, we may well enough suppose the *Gravity* equal throughout, and the descents of *Projects* in parallel lines, which in truth are towards the *Center*, the difference being so small as by no means to be discovered in *Practice*. The *Opposition* of the *Air*, 'tis true, is considerable against all light bodies moving through it, as likewise against small ones (of which more hereafter) but in great and ponderous Shot, this Impediment is found by *Experience* but very small, and may safely be neglected.

*Propositions concerning the Descent of heavy Bodies,
and the Motion of Projects.*

Prop. I. The *Velocities* of falling Bodies, are proportionate to the times from the beginning of their falls.

This follows, for that the action of *Gravity* being *continual*, in every space of time, the falling Body receives a new impulse, equal to what it had before, in the same space of time, received from the same power: For instance, in the first second of time, the falling Body has acquired a *Velocity*, which in that time would carry it to a certain distance, suppose 32 foot; and were there no new force, would descend at that rate with an *equable Motion*; but in the next second of time, the same power of *Gravity* continually acting thereon, superadds a new *Velocity* equal to the former; so that at the end of two seconds, the *Velocity* is double to what it was at the end of the first, and after the same manner may it be proved to be triple, at the end of the third second, and so on. Wherefore the *Velocities* of falling Bodies, are proportionate to the times of their falls, *Q. E. D.*

Prop. II. The *Spaces* described by the fall of a Body, are as the *Squares* of the times, from the beginning of the Fall.

Demonstration. Let *A B* (*Fig. 1. Tab. 1.*) represent the *time* of the fall of a Body, *BC* perpendicular to *A B* the *Velocity* acquired at the end of the fall, and draw the line *AC*, then divide the line *A B* representing the time into as many equal parts as you please, as *b, b, b, b, &c.* and through these points draw the lines *bc, bc, bc, bc, &c.* parallel to *BC*, 'tis manifest that the several lines, *bc*, represent the several *Velocities* of the falling Body, in such parts of the *time* as *A b* is of *A B*, by the former proposition. It is evident likewise that the *Area A B C* is the sum of all the lines *bc* being taken, according to the method of *Indivisibles*, infinitely many; so that
the

the *Area* ABC represents the sum of all the *Velocities*, between none and BC supposed infinitely many; which sum is the space descended in the time represented by AB. And by the same reason the *Areas* A b c, will represent the spaces descended in the times A b; so then the spaces descended in the times AB, Ab, are as the *Areas* of the *Triangles* A B C, A b c, which by the 20th of the 6 of *Euclid* are as the *Squares* of their *Homologous sides* A B, A b, that is to say, of the *Times*: wherefore the descents of *falling Bodies*, are as the *Squares* of the times of their fall, *Q. E. D.*

Prop. III. The *Velocity* which a *falling Body* acquires in any space of time, is double to that, wherewith it would have moved the space, descended by an equable motion, in the same time.

Demonstration, Draw the line EC parallel to AB and AE parallel to BC in the same *fig. 1.* and compleat the *Parallelogram* ABC E, it is evident that the *Area* thereof may represent the space, a *Body* moved equably with the *Velocity* BC, would describe in the time AB, and the *Triangle* ABC represents the *space* described by the *fall* of a *Body*, in the same time AB, by the second proposition. Now the *Triangle* ABC is half of the *Parallelogram* ABC E, and consequently the space described by the *fall*, is half what would have been described by an *equable Motion* with the *Velocity* BC, in the same time; wherefore the *Velocity* BC at the end of the *fall*, is double to that *Velocity*, which in the time AB, would have described the *space fallen*, represented by the *Triangle* ABC, with an *equable Motion*, *Q. E. D.*

Prop. IV. All *Bodies* on or near the surface of the *Earth*, in their *fall*, descend so, as at the end of the first second of time, they have described 16 feet one inch *London Measure*, and acquire d the *Velocity* of 32 feet two inches in a second.

This is made out from the 25th proposition of the second part of that Excellent Treatise of Mr. *Hugenius de Horologio Oscillatorio*; wherein he demonstrates the time of the least *Vibrations* of a *Pendulum*, to be to the time of the *fall* of a *Body*,
from

from the height of half the length of the *Pendulum*, as the *Circumference* of a *Circle* to its *Diameter*; whence as a *Corollary* it follows, that as the *Square* of the *Diameter* to the *Square* of the *Circumference*, so half the length of the *Pendulum* vibrating *seconds*, to the *space* described by the *fall* of a *body* in a *second* of *time*: and the length of the *Pendulum* vibrating *seconds*, being found 39,125, or $\frac{1}{8}$ Inches, the *descent* in a *second* will be found by the aforesaid *Analogy* 16 Foot and one Inch, and by the third *Proposition*, the *Velocity* will be double thereto; and near to this it hath been found by several *Experiments*, which by reason of the *swiftness* of the *fall*, cannot so exactly determine its *quantity*. The *Demonstration* of *Hugenius* being the *Conclusion* of a long train of *Consequences*. I shall for brevity sake omit; and refer you to his *Book*, where these things are more amply treated of.

From these four *Propositions*, all *Questions* concerning the *Perpendicular fall* of *bodies*, are easily *solved*, and either *Time*, *Height*, or *Velocity* being assigned, one may readily find the other two. From them likewise is the *Doctrine* of *Projects* deducible, assuming the two following *Axioms*; *viz.* That a *body* set a moving, will move on continually in a *right line* with an *equable motion*, unless some other force or impediment intervene, whereby it is accelerated, or retarded, or deflected.

Secondly, That a *Body* being agitated by two *motions* at a time, does by their *compounded forces* pass through the same *points*, as it would do, were the two *motions* divided and acted *successively*. As for instance, Suppose a *body* moved in the *Line* GF, (*Fig. 2. Tab. 1.*) from G to R, and there stopping, by another *impulse* suppose it moved in a *space* of *time* equal to the former, from R towards K, to V. I say, the *body* shall pass through the point V, tho' these two *several forces*, acted both in the *same time*.

Prop. V. The *Motion* of all *Projects* is in the *Curve* of a *Parabola*: Let the *line* GRF (*in fig. 2.*) be the *line* in which the *Project* is directed, and in which by the first *Axiom* it would

move equal *spaces* in equal *times*, were it not deflected downwards by the force of *Gravity*. Let *GB* be the *Horizontal line*, and *GC* a *Perpendicular* thereto. Then the *line GRF* being divided into equal parts, answering to equal *spaces* of *time*, let the *descents* of the *Project* be laid down in *lines parallel* to *GC*, proportioned as the *squares* of the *lines GS, GR, GL, GF*, or as the *squares* of the *times*, from *S* to *T*, from *R* to *V*, from *L* to *X*, and from *F* to *B*, and draw the *lines TH, VD, XY, BC* parallel to *GF*; I say the *Points T, V, X, B*, are *Points* in the *Curve* described by the *Project*, and that that *Curve* is a *Parabola*. By the second *Axiom* they are *Points* in the *Curve*; and the parts of the *descent GH, GD, GY, GC*, = to *ST, RV, LX, FB*, being as the *squares* of the *times* (by the second *Prop.*) that is, as the *squares* of the *Ordinates, HT, DU, YX, BC*, equal to *GS, GR, GL, GF*, the *spaces* measured in those *times*; and there being no other *Curve* but the *Parabola*, whose parts of the *Diameter* are as the *squares* of the *Ordinates*, it follows that the *Curve* described by a *Project*, can be no other than a *Parabola*: And saying, as *RU* the *descent* in any *time*, to *GR* or *UD* the *direct motion* in the same *time*, so is *UD*, to a *third* proportional; that *third* will be the *line* called by all *Writers* of *Conicks*, the *Parameter* of the *Parabola* to the *Diameter GC*, which is always the same in *Projects* cast with the same *Velocity*: And the *Velocity* being defined by the number of *feet* moved in a *second* of *time*, the *Parameter* will be found by dividing the *square* of the *Velocity*, by 16 *feet 1 inch*, the *fall* of a *body* in the same *time*.

Lemma.

The *Sine* of the double of any *Arch*, is equal to twice the *Sine* of that *Arch* into its *Co-sine*, divided by *Radius*; and the *Versed sine* of the double of any *Arch* is equal to the *square* of the *Sine* thereof divided by *Radius*.

Let the *Arch BC* (in *fig. 3.*) be double the *Arch BF*, and *A* the *Center*; draw the *Radii AB, AF, AC*, and the *Chord BDC*,

BDC, and let fall BE perpendicular to AC, and the *Angle* EBC, will be equal to the *Angle* ABD, and the *Triangle* BCE, will be like to the *Triangle* BDA; wherefore it will be as AB to AD, fo BC or twice BD; to BE, that is as *Radius* to *Co-sine*, fo twice *Sine*, to *Sine* of the double *Arch*. And as AB to BD, fo twice BD or BC, to EC, that is as *Radius* to *Sine*, fo twice that *Sine* to the *Versed-sine* of the double *Arch*; which two *Analogies* resolved into *Equations*, are the *Propositions* contained in the *Lemma* to be proved.

Prop. VI. The *Horizontal* distances of *Projections* made with the same *Velocity*, at several *Elevations* of the *Line* of direction, are as the *Sines* of the doubled *Angles* of *Elevation*.

Let GB (*fig. 2.*) the *Horizontal* distance be = z , the *sine* of the *Angle* of *Elevation*, FGB, be = s , its *Co-sine* = c , *Radius* = r , and the *Parameter* = p . It will be as c to s ; fo z to $\frac{s z}{c}$ = FB = GC, and by reason of the *Parabola* $\frac{p s z}{c}$ = to the *square* of CB, or GF, . Now as c to r , fo is z to $\frac{z r}{c}$ = GF, and

its *square* $\frac{z z r r}{c c}$ will be therefore = to $\frac{p s z}{c}$: which *Equation* re-

duced, will be $\frac{p s c}{r r}$ = z . But by the former *Lemma* $\frac{2 s c}{r}$ is

equal to the *Sine* of the double *Angle*, whereof s is the *Sine*: wherefore 'twil be as *Radius* to *Sine* of double the *Angle* FGB, fo is half the *Parameter*, to the *Horizontal rang* or *distance* fought; and at the several *Elevations*, the *ranges* are as the *sines* of the double *Angles* of *Elevation* Q.E.D.

Corollary.

Hence it follows, that half the *Parameter* is the greatest *Randon*, and that that happens at the *Elevation* of 45 degrees, the *sine* of whose double is *Radius*, . Likewise that the *Ranges* equally distant above and below 45 are equal, &c

are the *sines* of all doubled *Arches*, to the *sines* of their doubled *Complements*.

Prop. VII. The *Altitudes* of *Projections* made with the same *Velocity*, at several *Elevations*, are as the *versed sines* of the doubled *Angles* of *Elevation*: As c is to s :: so is $\frac{p\ s\ c}{r\ r}$
 $=$ GB to $\frac{p\ s\ s}{r\ r}$ $=$ BF, and $UK=RU= \frac{BF}{4}$, the *Altitude* of the *Projection* $= \frac{p\ s\ s}{4\ r\ r}$. Now by the foregoing *Lemma* $\frac{2\ s\ s}{r}$ $=$ to the *versed sine* of the double *Angle*, and therefore it will be as *Radius*, to *versed sine* of double the *Angle* FGB, so an 8th of *Parameter* s to the height of the *Projection* VK; and so these heights at several *Elevations* are as the said *versed sines*, Q. E. D.

Corollary.

From hence it is plain, that the greatest *Altitude* of the perpendicular *Projection* is a 4th of *Parameter*, or half the greatest *Horizontal Rang*; the *versed sine* of 180 degrees being $= 2\ r$.

Prop. VIII. The *Lines* GF, or times of the flight of a *Project* cast with the same degree of *velocity* at different *Elevations*, are as the *sines* of the *Elevations*.

As c is to r :: so is $\frac{p\ s\ c}{r\ r} =$ GB by the 6 *Prop.* to $\frac{p\ s}{r} =$ GF, that is as *Radius* to *sine* of *Elevation*, so the *Parameter* to the *line* GF; so the *lines* GF are as the *sines* of *Elevation*, and the *Times* are proportional to those *Lines*; wherefore the *Times* are as the *Sines* of *Elevation*: Ergo constat propositio.

Prop. IX. Problem. A *Projection* being made as you please, having the *Distance* and *Altitude*, or *Descent* of an *Object*, through which the *Project* passes, together with the *Angle* of *Elevation* of the *line* of *Direction*; to find the *Parameter* and *Velocity*, that is (in *Fig. 2.*) having the *Angle* FGB, GM, and MX.

Solution. As *Radius* to *Secant* of FGB, so GM the *distance* given

given, to GL; and as *Radius* to *Tangent* of FGB, so GM to LM. Then LM—MX in *heights*, or +MX in *descents*; or else MX—ML, if the *direction* be below the *Horizontal-line*, is the *fall* in the *time* that the direct *impulse* given in G would have carried the *Project* from G to L=LX=GY; then by reason of the *Parabola*; as LX or GY, is to GL or YX, :: fo is GL to the *Parameter* sought. To find the *Velocity* of the *Impulse*, by Prop.2. & 4, find the *time* in *seconds* that a *body* would fall the *space* LX, and by that dividing the *line* GL, the *Quote* will be the *Velocity*, or *space* moved in a *second* sought, which is alwaies a *mean proportional* between the *Parameter* and 16 feet 1 inch.

Prop. X. Problem 2. Having the *Parameter*, *Horizontal distance*, and *height* or *descent* of an *Object*, to find the *Elevations* of the *line of direction* necessary to hit the given *Object*; that is, having GM, MX, and the greatest *Random* equal to half the *Parameter*; to find the *Angles* FGB.

Let the *Tangent* of the *Angle* sought be =*t*, the *Horizontal distance* GM=*b*, the *Altitude* of the *Object* MX=*b*, the *Parameter*=*p*, and *Radius*=*r*, and it will be,

As *r* to *t*, so *b* to $\frac{tb}{r}$ =ML and $\frac{t}{r} + b$ in ascents =LX, and $\frac{p}{r} + b$ in descents =LX, and $\frac{p}{r} + b$ =GL quad. =XY quad. racione *Parabola*; but

$\frac{tb}{r} + \frac{tt}{r} = \frac{tb}{r} + \frac{tt}{r} = \text{GL quad. 47. I. Euclid.}$ Wherefore

$\frac{p}{r} + b = \frac{tb}{r} + \frac{tt}{r}$ which Equation transposed, is

$\frac{tt}{r} = \frac{p}{r} + b - \frac{tb}{r}$, divided by *b b* is

$\frac{tt}{r} = \frac{p}{r} + \frac{pb}{bb}$ — 1. this Equation shews the *Question* to

have two Answers, and the Roots thereof are $\frac{t}{r} = \frac{p}{2b} \mp$

$\sqrt{\frac{pp + 4pb}{4bb}}$ — 1 from which I derive the following Rule.

Divide half the *Parameter* by the *Horizontal* distance, and keep the *Quote* ; viz. $\frac{p}{2b}$ then say, as *square* of the *distance*

given to the half *Parameter*, so half *Parameter* \mp double height
descent to the *square* of a *Secant* = $\frac{pp \mp 4pb}{4bb}$

the *Tangent* answering to that *Secant*, will be $\sqrt{\frac{pp \mp 4pb}{4bb} - 1}$

or rr : so then the sum and difference of the afore-found *Quote*, and this *Tangent* will be the *Roots* of the *Equation*, and the *Tangents* of the *Elevations* sought.

Note here, that in *Descents*, if the *Tangent* exceed the *Quote*, as it does when ph is more than bb , the *direction* of the lower *Elevation* will be below the *Horizon*, and if $ph=bb$, it must be directed *Horizontal*, and the *Tangent* of the upper *Elevation* will be $\frac{p}{b}r$: Note likewise, that if $4bb+4ph$ in *ascents*, or $4bb-4ph$ in *descents*, be equal to pp , there is but one *Elevation* that can hit the *Object*, and its *Tangent* is $\frac{p}{2b}r$ and if $4bb+4ph$ in *ascents*, or $4bb-4ph$ in *descents*, do exceed pp , the *Object* is without the reach of a *Project* cast with that *Velocity*, and so the thing impossible.

From this *Equation* $4bb \mp 4ph = pp$ are determined the utmost limits of the reach of any *Project*, and the *Figure* assigned, wherein are all the *heights* upon each *Horizontal* distance beyond which it cannot pass ; for by reduction of that *Equation*, h will be found = $\frac{1}{2}p - \frac{bb}{p}$ in *heights*, and $\frac{bb}{p} - \frac{1}{2}p$ in *descents* ; from whence it follows, that all the *Points* h are in the *Curve* of the *Parabola*, whose *Focus* is the *Point* from whence the *Project* is cast, and whose *Latus rectum*, or *Parameter ad Axem* is $=p$. Likewise from the same *Equation* may the least *Parameter* or *Velocity* be found capable to reach the *Object*

Object proposed; for $bb = \frac{1}{2}pp \mp ph$ being reduced $\frac{1}{2}p$ will be $= \sqrt{bb + hb} \pm b$ in ascents, which is the *Horizontal* *rang* at 45 degrees, that would just reach the *Object*, and the *Elevation* requisite will be easily had; for dividing the so found *Semi-parameter* by the *Horizontal distance* given b , the *Quote* into *Radius* will be the *Tangent* of the *Elevation* sought. This Rule may be of good use to all *Bombardiers* and *Gunners*, not only that they may use no more Powder than is necessary, to cast their *Bombs* into the place assigned, but that they may shoot with much more certainty, for that a small Error committed in the *Elevation* of the *Piece*, will produce no sensible difference in the fall of the Shot: For which Reasons the *French Engineers* in their late Sieges have used *Morter-pieces* inclined constantly to the *Elevation* of 45, proportioning their Charge of Powder according to the distance of the *Object* they intend to strike on the *Horizon*.

And this is all that need to be said concerning this *Problem*, of Shooting upon *heights* and *descents*. But if a *Geometrical* construction thereof be required; I think I have one, that is as easy as any can be expected, which I deduce from the forgoing *Analytical Solution*, viz. $\frac{t}{r} = \frac{p}{2b} \pm \frac{\sqrt{\frac{1}{2}pp \pm ph - b^2}}{bb}$, and tis this. Having made the right *Angle* LDA , *Tab. I. fig. 4.* make $DA, DF = p$, or greatest *Rang*, $DG = b$ the *Horizontal* distance, and $DB, DC = b$, the *Perpendicular height* of the *Object*; and draw GB , and make $DE =$ thereto. Then with the *Radius* AC and *center* E sweep an Arch, which if the thing be possible, will Intersect the line AD in H ; and the line DH being laid both waies from F will give the points K and L , to which draw the lines GL, GK ; I say the *Angles* LGD, KGD are the *Elevations* required for hitting the *Object* B . But note that if B be below the *Horizon*, its *descent* $DC = DB$ must be laid from A , so as to have $AC =$ to $AD + DC$. Note likewise, that if in *descents* DH be greater than FD , and so K fall below D the

Angle

Angle KGD shall be the depression below the *Horizon* : Now this Construction so naturally follows from the *Equation*, that I shall need say no more about it.

Prop. XI. To determine the force or *Velocity* of a *Project*, in every point of the *Curve* it describes.

To do this we need no other *pracognita*, but only the third Proposition, *Viz.* that the *Velocity* of *falling Bodies*, is double to that which in the same time, would have described the space *fallen* by an equable motion : For the *Velocity* of a *Project*, is compounded of the constant equal *Velocity* of the impressed motion, and the *Velocity* of the *fall*, under a given *Angle*, *viz.* the complement of the *Elevation* : For instance, in *Fig. 2*, in the time wherein a project would move from G to L, it descends from L to X, and by the third *Proposition* has acquired a *Velocity*, which in that time would have carried it by an equable motion from L to Z or twice the descent L X; and drawing the line G Z, I say the *Velocity* in the point X, compounded of the *Velocities* G L and L Z under the *Angle* G L Z, is to the *Velocity* imprest in the point G, as G Z is to G L; this follows from our second *Axiome*; and by the 20 and 21. *Prop. lib. 1, conic. Midorgii*, XO parallel and equal to G Z shall touch the *Parabola* in the point X. So that the *Velocities* in the several points, are as the lengths of the *Tangents* to the *Parabola* in those points, intercepted between any two *Diameters* : And these again are as the *Secants* of the *Angles*, which those *Tangents* continued make with the *Horizontal* line G B. From what is here laid down, may the comparative force of a *shot* in any two points of the *Curve*, be either *Geometrically* or *Arithmetically* discovered.

Corollary.

From hence it follows, that the force of a *Shot* is alwaies least at U, or the *Vertex* of the *Parabola*, and that at equal distances therefrom, as at T and X, G and B its force is alwaies equal, and that the least force in U is to that in G and B, as

B, as *Radius* to the *Secant* of the *Angle of Elevation* F G B.

These *Propositions* considered, there is no question relating to *Projects*, which by the help of them may not easily be Solved; and tho' it be true that most of them are to be met withal, in *Galileus*, *Toricellius* and others, who have taken them from those Authors, yet their Books being Forreign, and not easy to come by, and their *Demonstrations* long and difficult, I thought it not amiss to give the whole *Doltrine* here in *English*, with such short *Analytical* Proof of my own, as might be sufficient to evince their Truth.

The *Fenth Proposition* contains a *Problem*, untouch't by *Toricellius*, which is of the greatest use in *Gunnery*, and for the sake of which this *Discourse* was principally intended; It was first Solved by Mr. *Anderson*, in his Book of the Genuine use and effects of the *Gunn*, Printed in the Year 1674; but his Solution required so much Calculation, that it put me upon search, whether it might not be done more easily, and thereupon in the Year 1678 I found out the rule I now publish, and from it the *Geometrical* Construction: Since which time there has a large *Treatise* of this Subject Entituled, *L'art de jeter les Bombes*, been Published in *France* by *Monsieur Blondel*, wherein he gives the Solutions of this *Problem* by *Messieurs Bout, Romer* and *de la Hire*; But none of them being the same with mine, or in my Opinion more easy, and most of them more Operose, and besides mine finding the *Tangent*, which generally determines the *Angle* better than its *Sine*, I thought my self obliged to Print it for the use of all such, as desire to be informed in the *Mathematical* part, of the Art of *Gunnery*.

Now these rules were rigidly true, were it not, as I said before, for the Opposition of the Medium, whereby not only the direct impress Motion is continually retarded, by likewise the increase of the *Velocity* of the *fall*, so that the spaces described thereby, are not exactly as the squares of the times: But what this Opposition of the *Air* is, against several *Velocities*, *Bulks*, and *Weigats*, is not so easy to determine. This

certain that the weight of *Air*, to that of *Water*, is nearly as 1 to 800, whence the weight thereof, to that of any *Project* is given; tis very likely, that to the same *Velocity*, and *Magnitude*, but of different matter, the *Opposition* should be *reciprocally* as the weights of the shott; as likewise that to shott of the same *Velocity* and matter, but of different Sizes, it should be as the *Diameters reciprocally*: whence generally the *Opposition* to shott with the same *Velocity*, but of differing *Diameters*, and *Materials*, should be as their *Specifick Gravities* into their *Diameters reciprocally*; but whether the *Opposition*, to differing *Velocities* of the same shott, be as the *Squares* of those *Velocities*, or as the *Velocities* themselves, or otherwise, is yet a harder Question. However it be, tis certain, that in large Shott of *Mettal*, whose weight many Thousand times Surpasses that of the *Air*, and whose force is very great, in proportion to the *Surface* wherewith they press thereon; this *Opposition* is scarce discernable: For by several *Experiments* made with all Care and Circumspection with a *Morterpeice* Extraordinary well fixt to the *Earth* on purpose, which carried a Solid Brass Shott of 4½ Inches *Diameter*, and of about 14 Pound weight, the *Ranges* above and below 45 *Degrees* were found nearly equal; if there were any difference, the under *Ranges* went rather the farthest, but those differences were usually less than the Errours committed in ordinary *Practice*, by the unequal Goodness and Dryness of the same sort of *Powder*, by the Unfitness of the *Shott* to the *Bore*, and by the Loosness of the Carriage.

In a Smaller *Brass-Shott* of about an Inch and half *Diameter*, cast by a *Cross-Bow* which *ranged* it, at most about 400 *foot*, the Force being much more Equal than in the *Morterpeice*, this difference was found more Curiously, and Constantly and most Evidently, the under *Ranges* out went the upper. From which Trials I conclude, that altho' in small and light *Shott*, the *Opposition* of the *Air*, ought and must be accounted for; yet in *Shooting* of Great and Weighty *Bombs*, there need be very little or no allowance made; and so these *Rules* may be

be put in *Practice* to all Intents and Purposes, as if this *Impediment* were absolutely Removed.

*An Account of an Experiment shewn before the Royal Society, of Shooting by the Rarefaction of the Air :
By Dr. D. Papin, R. S. S.*

WHereas ordinary Wind-Guns do their Effect by the Compression of the Air. *Otto Ghericke* hath found a new Sort that shoots by Rarefaction ; and he hath Published that device at large in his Book about *Pneumatick Experiments*, but he doth not express how strong was the Effect. I have therefore had the Curiosity to try it my self by another Contrivance, which I take to be better than his : First, because I can make a Rarefaction much more perfect than he could do. Secondly, because his Device could not be used but for Guns of a small bore ; but my way may be apply'd to the biggest bore that can be made by Workmen : So that one might by this means throw up vast Weights to a great distance.

A A is a Pipe very equal from one end to the other.

BB a small Pipe foder'd to a Hole near the end of the Pipe A A, and apply'd to the Plate of the *Pneumatick Engine*.

CCCC some kind of Stool to bear up the hinder part of the Pipe A A.

D. a peice of Lead fitted to the bore of the pipe A A.

The pipe A A is to be shut at both ends by *Valves* outwardly apply'd, and so the said pipe A A, though never so big, may be exhausted of Air by means of the *Pneumatick Engine* : Which done, the *Valve* towards D must be suddenly open'd, so that the whole pressure of the *Atmosphere* acting upon the Lead D may drive it along the pipe A A with

Tab 1.

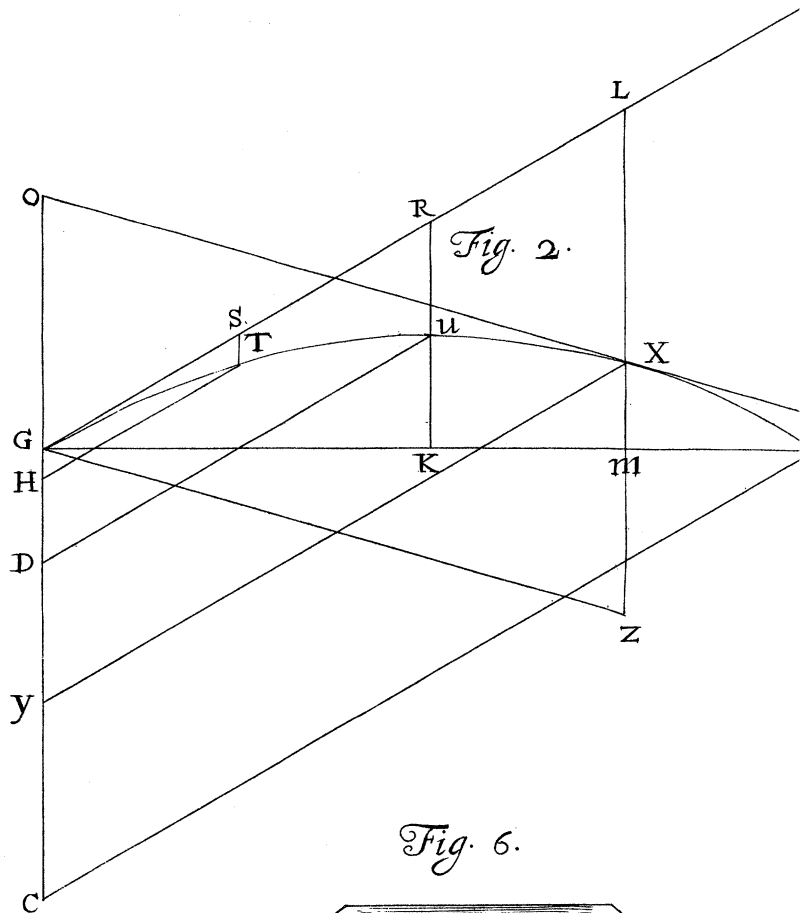
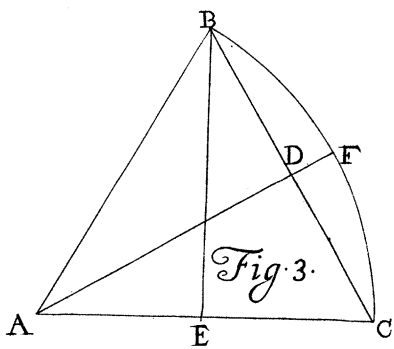
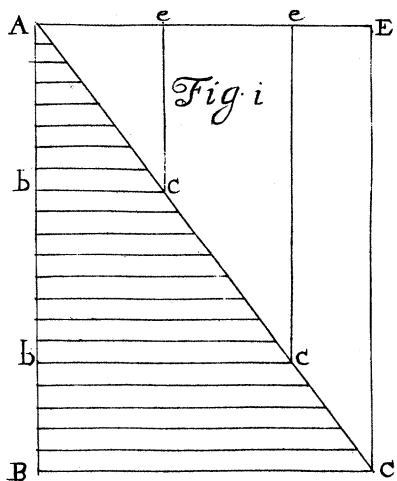


Fig. 6.

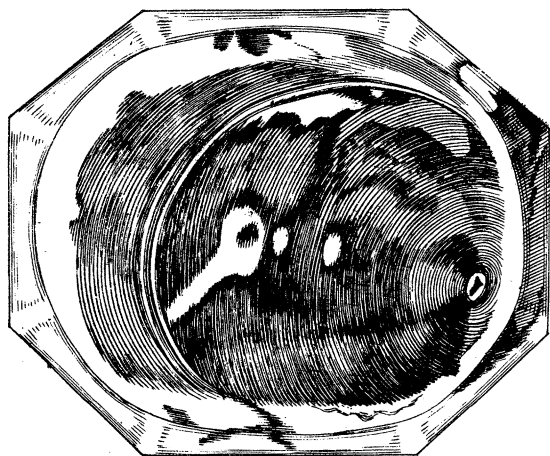
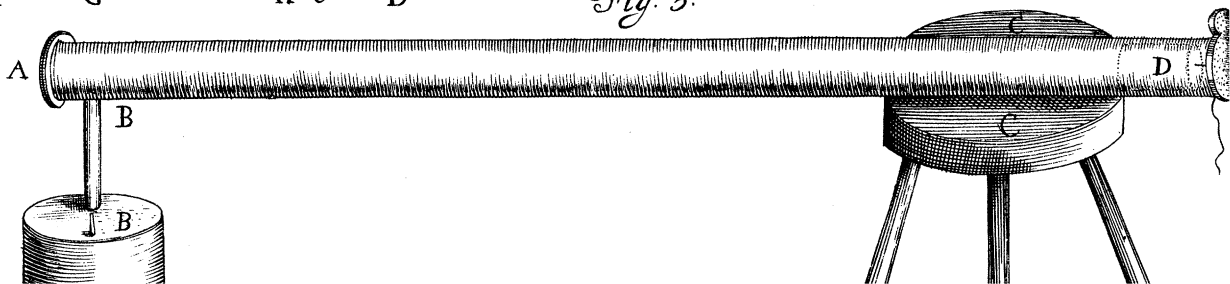
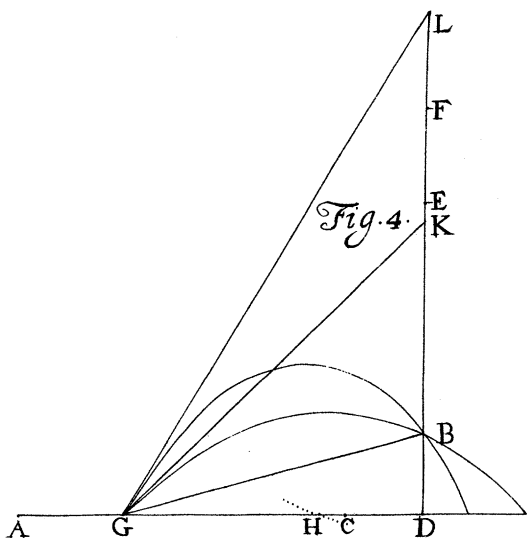
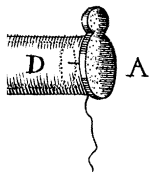
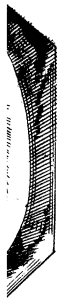
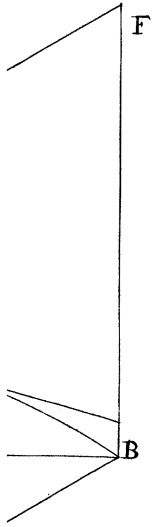
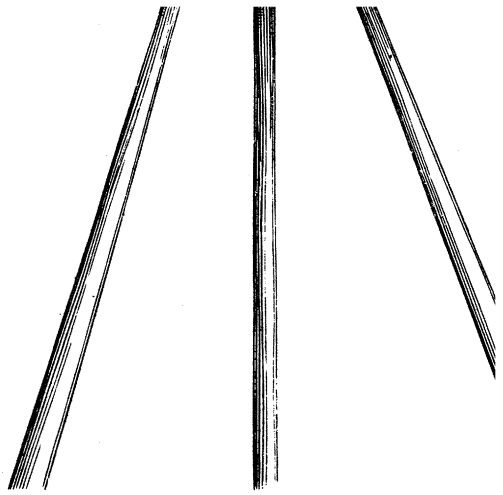
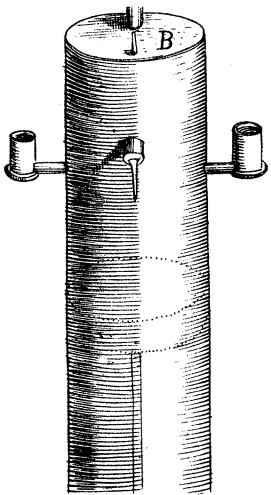
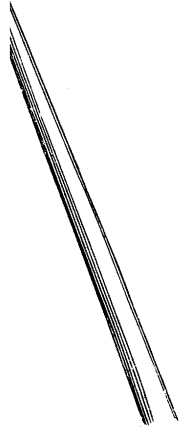


Fig. 5.









Tab 1.

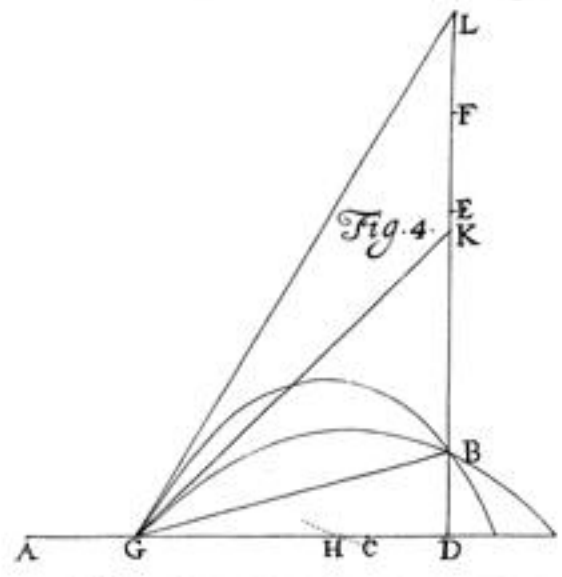
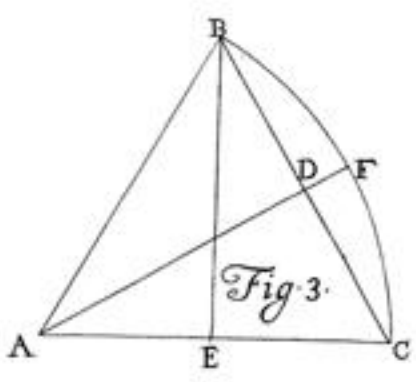
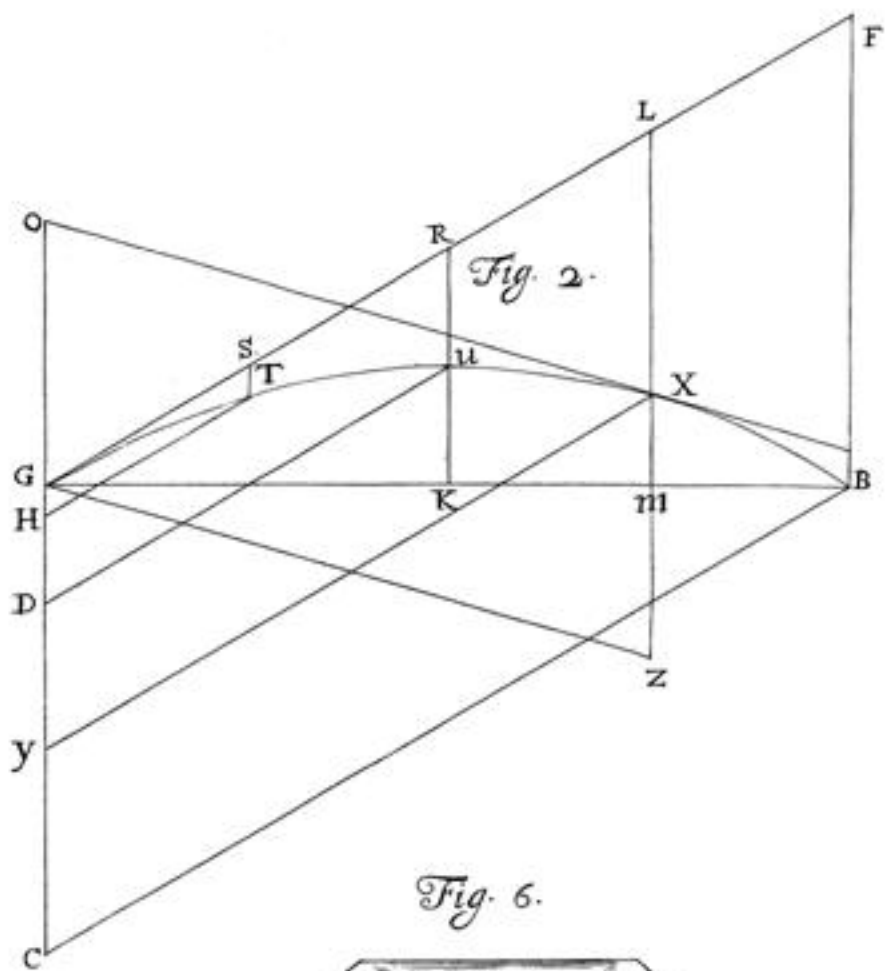
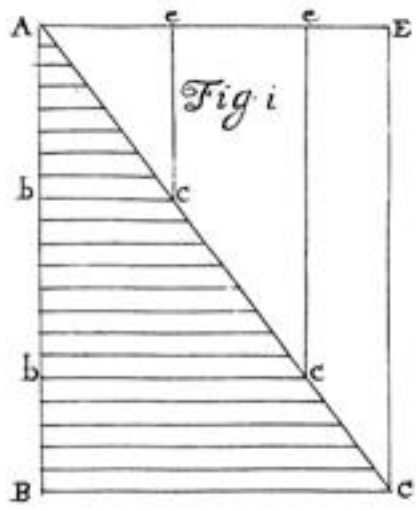


Fig. 5.

